Reader's Forum

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Comment on "Postbuckling Behavior of Laminated Plates Using a Direct Energy-Minimization Technique"

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MINGUET et. al.[1] presented an energy-minimization technique for the postbuckling analysis of laminated plates. There the Reissner-Mindlin theory of plates[2,3] has been used. The following observations may be made with regard to the displacement boundary conditions used in the case of a clamped boundary and the consequent constraining of the problem. This constraint may be only of theoretical importance as this plate theory itself constrains the normals to the middle plane to remain straight after deformation, resulting in constant transverse shear strains at a cross section and, in the case of a composite plate, also producing a discontinuity of the transverse shear stress at layer interfaces.

Consider the plate boundaries parallel to the y axis and the displacement approximation in the x coordinate. The displacement variables ω and ψ have been expressed in this longitudinal direction by the use of the classical beam functions F_i of a clamped beam as the bases. The variable ϕ has been considered as a series of the derivatives F_i' of the same beam functions. These series have been used on the assumption that the lateral displacement ω, its derivative normal to the boundary $\omega_{,x}$, and the normal rotation ϕ should vanish at a clamped boundary. This use of three boundary conditions constrains the problem as the particular plate theory needs only the vanishing of the lateral displacement and the normal rotation.[3,4] Although only the displacement boundary conditions need to be satisfied in a Rayleigh-Ritz analysis, the constraint implicitly assumes that the normal tranverse shear strain is zero at the clamped boundary. This occurs in the strain ϵ_5 due to the vanishing of both the normal rotation ϕ and the normal derivative of ω on their own. Considering the fact that the parallel transverse shear strain ϵ_4 also is zero at the boundary, this further implies the nonexistence of transverse shear stresses along the support even in an anisotropic plate where the two transverse shear strains are coupled. Here it may be mentioned that the transverse shear stresses are usually computed by the use of the equilibrium equations, instead of the constitutive equation, due to the constant values of the transverse shear strains that the theory produces.

The solution to this theoretical predicament lies in the use of the beam functions for a Timoshenko beam, as suggested by Dawe.[5] These come in pairs for the lateral displacement and the normal rotation and help to have the proper boundary conditions. The only problem in this approach is that one has to solve for the modes for each new problem, as the results do not occur in closed form.

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Reply by Authors to T.D.G. Canisius

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THE authors wish to express their thanks to Canisius for drawing attention to the clamped boundary conditions used in the buckling analysis of the sandwich plate.[1] This was discussed in more detail in the longer report[2] on which the article was based. Since the actual sandwich plate was clamped by filling in the ends with epoxy resin inside a narrow slit at the boundary, the sides of the slit and the stiffer faces prevented angular shearing deflections at the immediate edges as might have occurred if the edges were simply butted up against a rigid wall. The deflections did appear to have zero slope as well as zero deflection and zero rotation angle at the boundary, and hence the mode shapes used for the Rayleigh-Ritz analysis seemed to fit the geometrical boundary conditions of the problem as desired. As mentioned by Canisius, the transverse shear stresses themselves were then calculated using the equilibrium equations rather than the constitutive equations of the sandwich plate so that finite shear stresses were obtained at the boundaries.

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